

DALITZ PLOT STRUCTURE IN $D^0 \rightarrow \pi^+\pi^-\pi^0$ Bhubanjyoti Bhattacharya^{a,1}, Cheng-Wei Chiang^{b,c,2} and Jonathan L. Rosner^{a,3}^a *Enrico Fermi Institute and Department of Physics
University of Chicago, 5640 S. Ellis Avenue, Chicago, IL 60637*^b *Department of Physics and Center for Mathematics and Theoretical Physics
National Central University, Chungli, Taiwan 320, Republic of China*^c *Institute of Physics, Academia Sinica
Taipei, Taiwan 115, Republic of China*

The BaBar Collaboration has pointed out that $D^0 \rightarrow \pi^+\pi^-\pi^0$ is dominated by an isospin-zero final state, leading to nearly complete depletion of the Dalitz plot along all three diagonals. In flavor-SU(3) approaches to charmed particle decays to a light vector and a light pseudoscalar particle, this behavior is seen, but does not appear to have a fundamental origin. Instead, it arises as a result of approximate cancellation of higher-isospin combinations of several types of amplitudes: color-favored tree, color-suppressed tree, and exchange. Interpretation in terms of a direct-channel effect would require an exotic resonance, with spin, parity, and charge-conjugation eigenvalues $J^{PC} = 0^{--}$.

PACS categories: 11.30.Hv, 12.39.St, 13.25.Ft, 14.40.Lb

I INTRODUCTION

A curious feature of the decay $D^0 \rightarrow \pi^+\pi^-\pi^0$ has been pointed out by the BaBar Collaboration [1, 2, 3]. Although states of isospin zero, one, and two are possible in principle, the Dalitz plot shows strong depopulation along each of its symmetry axes, characteristic of a final state with isospin zero [4]. In the present paper we show that this behavior is expected in a flavor-SU(3) analysis based on a graphical language [5]. It validates certain assumptions made in analyzing decays of charmed mesons within that language, including factorization [6, 7, 8]. However, it still invites explanation at a deeper level.

We review the isospin decomposition of three-pion amplitudes in Section II and the graphical description of charmed meson decays in Section III. We then apply results obtained in flavor-SU(3) fits to charmed meson decay rates to the construction of isospin amplitudes in Section IV, finding dominance of $I = 0$ as measured experimentally [1, 2, 3]. We discuss reasons for this agreement in Section V and conclude in Section VI.

¹bhujyo@uchicago.edu²chengwei@ncu.edu.tw³rosner@hep.uchicago.edu

II ISOSPIN DECOMPOSITION

We retrace steps noted in Refs. [1] and [2], whose conventions are slightly different from ours. In accord with standard usage for Clebsch-Gordan coefficients [9], we define

$$\rho^+ = \frac{1}{\sqrt{2}}[\pi^+\pi^0 - \pi^0\pi^+] , \quad (1)$$

$$\rho^0 = \frac{1}{\sqrt{2}}[\pi^+\pi^- - \pi^-\pi^+] , \quad (2)$$

$$\rho^- = \frac{1}{\sqrt{2}}[\pi^0\pi^- - \pi^-\pi^0] . \quad (3)$$

The BaBar papers mentioned above choose the opposite sign for ρ^0 .

With our definitions the isospin amplitudes (again in accord with standard usage [9]) are

$$A_0 = \frac{1}{\sqrt{3}}[\rho^+\pi^- - \rho^0\pi^0 + \rho^-\pi^+] , \quad (4)$$

$$A_1 = \frac{1}{\sqrt{2}}[\rho^+\pi^- - \rho^-\pi^+] , \quad (5)$$

$$A_2 = \frac{1}{\sqrt{6}}[\rho^+\pi^- + 2\rho^0\pi^0 + \rho^-\pi^+] , \quad (6)$$

where we have used $\rho^i\pi^j$ as a shorthand for $\mathcal{A}(D^0 \rightarrow \rho^i\pi^j)$.

A purely isospin-zero spin-zero configuration of three pions consists of each pair coupled to isospin one so as to couple with the third pion to isospin zero. Because of Bose statistics, each pion pair must then be antisymmetric under interchange of the pions. The matrix element must then be of the form

$$\mathcal{A}(D^0 \rightarrow \pi^+\pi^-\pi^0) = f(\mathbf{p}_+, \mathbf{p}_-, \mathbf{p}_0) , \quad (7)$$

where f is a totally antisymmetric function of its arguments and \mathbf{p}_i is the three-momentum of pion π^i in the D^0 center-of-mass system (c.m.s.) [4]. The corresponding Dalitz plot must have no events along each of its symmetry axes.

If $A_1 = A_2 = 0$, one must have

$$\mathcal{A}(D^0 \rightarrow \rho^+\pi^-) = -\mathcal{A}(D^0 \rightarrow \rho^0\pi^0) = \mathcal{A}(D^0 \rightarrow \rho^-\pi^+) . \quad (8)$$

Figs. 1 and 2 then permit one to see the destructive interference along each symmetry axis of the Dalitz plot.

The (horizontal, vertical, diagonal) bands respectively denote the (ρ^+, ρ^-, ρ^0) regions. Because each ρ decays to $\pi\pi$ in a P-wave, the signs of the $\rho\pi$ amplitudes are opposite at opposite ends of each band. Bands and labels of each of their ends correspond to configurations shown in Fig. 2.

With decay amplitudes obeying Eq. (8), and ρ decays described by Eqs. (1–3), each overlap region contains one amplitude of one sign and another of the opposite sign, symptomatic of a vanishing amplitude along each symmetry axis of the Dalitz plot. Indeed, the BaBar Collaboration's amplitudes shown in Table I, where we have expressed the $\rho^0\pi^0$ amplitude in our phase convention, approximately satisfy Eq. (8), leading to dominance of the $I = 0$ amplitude as noted in Table II. We also plot in Fig. 3 the magnitudes and phases of the decay amplitudes for $\rho(770)\pi$ charge states and isospins, in a phase convention where A_0 is real and negative.

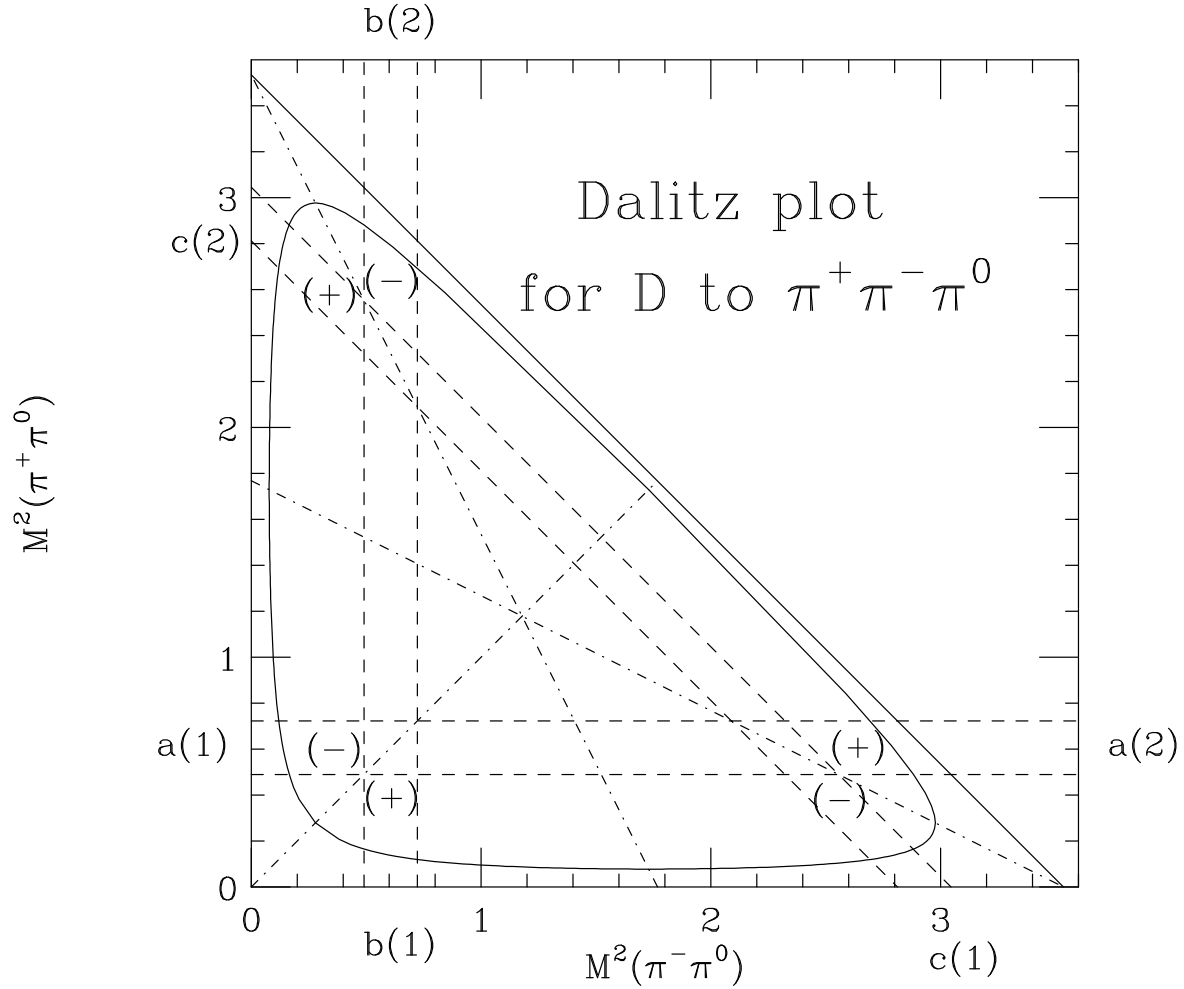


Figure 1: Dalitz plot for $D^0 \rightarrow \pi^+ \pi^- \pi^0$ illustrating $\rho(770)$ bands (between pairs of dashed lines), symmetry axes (dash-dotted lines) along which $I = 0$ amplitudes vanish, and relative signs of interfering amplitudes in regions where bands cross. Bands are labeled by letters (a,b,c) for (ρ^+, ρ^-, ρ^0) ; the two ends of each band are labeled by numbers corresponding to the configurations illustrated in Fig. 2.

Table I: Observed amplitudes (arbitrary overall normalization) and phases for $D^0 \rightarrow \rho(770)\pi$ [3]. We do not show several other amplitudes in the fit, all of which have fractions less than a few percent. Our convention for the phase of $A(\rho^0 \pi^0)$ differs from that of Refs. [1-3] by 180° .

Channel	Amplitude	Phase ($^\circ$)	Fraction (%)
$\rho(770)^+ \pi^-$	$0.823 \pm 0.000 \pm 0.004$	0 (def.)	$67.8 \pm 0.0 \pm 0.6$
$\rho(770)^0 \pi^0$	$0.512 \pm 0.005 \pm 0.011$	$-163.8 \pm 0.6 \pm 0.4$	$26.2 \pm 0.5 \pm 1.1$
$\rho(770)^- \pi^+$	$0.588 \pm 0.007 \pm 0.003$	$-2.0 \pm 0.6 \pm 0.6$	$34.6 \pm 0.8 \pm 0.3$

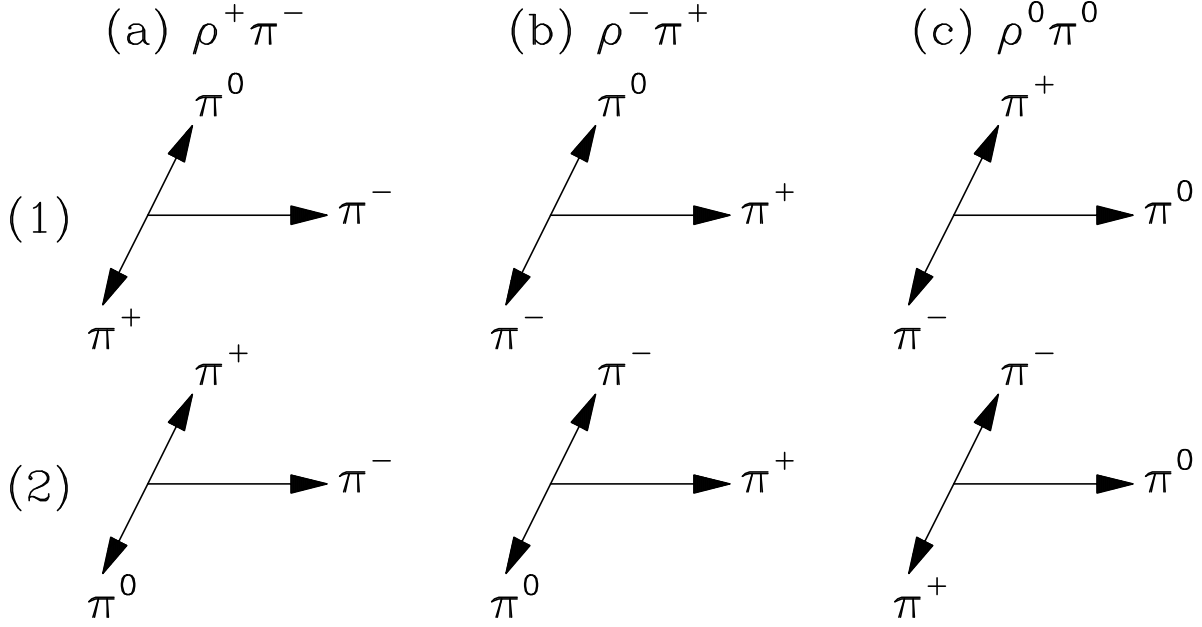


Figure 2: Configurations in $D^0 \rightarrow \rho\pi$ corresponding to regions in Fig. 1. Each configuration is illustrated in the ρ c.m.s. When one of the decay products of the ρ makes a small angle with the bachelor pion, it is near the kinematic region in which the two can form another ρ .

Table II: Observed isospin amplitudes and phases for $D^0 \rightarrow \rho(770)\pi$, based on Clebsch-Gordan coefficients in Eqs. (4)–(6) and amplitudes quoted in Table I. We have normalized amplitudes so that the sum of their absolute squares is 1. These are not the same as the isospin amplitudes quoted in Ref. [3], which include contributions from higher ρ states.

Channel	Amplitude	Phase ($^\circ$)	Fraction (%)
$I = 0$	0.9708 ± 0.0021	0 (def.)	94.24 ± 0.40
$I = 1$	0.1474 ± 0.0057	1.3 ± 2.0	2.17 ± 0.17
$I = 2$	0.1893 ± 0.0076	-39.3 ± 13.0	3.58 ± 0.29

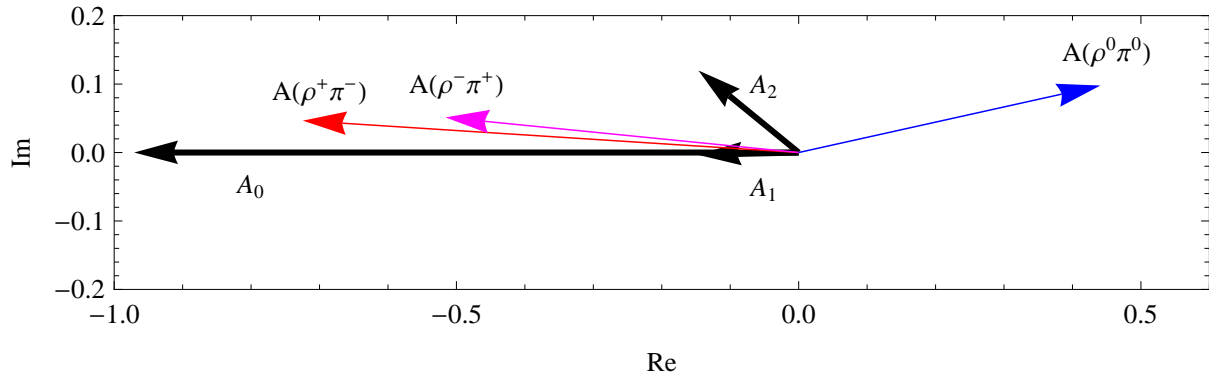


Figure 3: Amplitudes and phases for $D^0 \rightarrow \rho(770)\pi$ charge states and isospins observed in the BaBar analysis [3]. Amplitudes have been normalized so that the sums of their squares (for charge states or isospins) is 1, and A_0 has been taken real and negative.

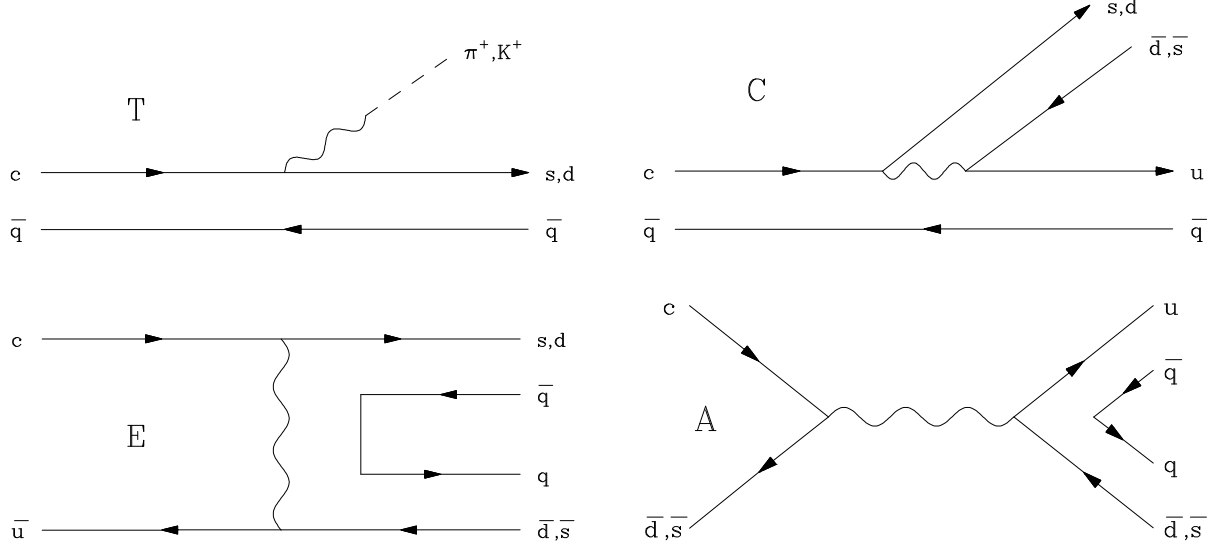


Figure 4: Flavor topologies for describing charm decays. T : color-favored tree; C : color-suppressed tree; E exchange; A : annihilation. The $D^0 \rightarrow \pi^+ \pi^- \pi^0$ decays considered here involve the CKM matrix elements $V_{cd}^* V_{ud}$ in graphs T , C , and E .

III GRAPHICAL AMPLITUDE DESCRIPTION

Within the context of a graphical notation equivalent to flavor SU(3) [5], decays of charmed mesons to pairs of light mesons have been analyzed recently [6, 7, 8]. For decays to a pseudoscalar meson P and a vector meson, amplitudes are labeled by the type of diagram (“tree” T , “color-suppressed” C , “exchange” E , and “annihilation” A), as shown in Fig. 4. A subscript P or V denotes the meson containing the spectator quark, and a prime denotes a Cabibbo-suppressed amplitude, defined here as related to the corresponding Cabibbo-favored amplitude by $\tan \theta_C = 0.2305$, where θ_C is the Cabibbo angle. (Ref. [10] quotes $\sin \theta_C = 0.2246 \pm 0.0012$.) The partial width is given in terms of the invariant amplitude \mathcal{A} as

$$\Gamma(D \rightarrow PV) = \frac{p^{*3}}{8\pi M_D^2} |\mathcal{A}|^2, \quad (9)$$

where p^* is the magnitude of the c.m.s. 3-momentum of each final particle. The relevant decay amplitudes are then

$$\mathcal{A}(D^0 \rightarrow \rho^+ \pi^-) = -(T'_P + E'_V), \quad (10)$$

$$\mathcal{A}(D^0 \rightarrow \rho^- \pi^+) = -(T'_V + E'_P), \quad (11)$$

$$\mathcal{A}(D^0 \rightarrow \rho^0 \pi^0) = \frac{1}{2}(E'_P + E'_V - C'_P - C'_V). \quad (12)$$

The magnitudes and phases of these amplitudes obtained in flavor-SU(3) fits to a large number of Cabibbo-favored D^0 , D^+ , and D_s decays to PV final states are summarized in Table III. Two different sets of values are quoted, corresponding to Refs. [7] and [8]. They differ mainly in their handling of η - η' mixing, leading to different values of E'_V , and to a lesser extent T'_P and C'_V . Each amplitude is quoted in units of 10^{-6} .

Table III: Amplitudes obtained in fits to Cabibbo-favored decays of charmed particles to PV final states, in units of 10^{-6} .

Amplitude	Ref. [7]		Ref. [8]	
	Magnitude	Phase ($^{\circ}$)	Magnitude	Phase ($^{\circ}$)
T'_P	1.719 ± 0.048	0 (def.)	1.776 ± 0.083	0 (def.)
T'_V	0.910 ± 0.016	0 (def.)	0.910 ± 0.036	0 (def.)
C'_V	0.797 ± 0.041	172 ± 3	0.909 ± 0.100	164 ± 23
C'_P	1.125 ± 0.035	-162 ± 1	1.126 ± 0.070	-162 ± 3
E'_V	0.546 ± 0.044	-110 ± 4	0.330 ± 0.183	-124 ± 41
E'_P	0.678 ± 0.021	-93 ± 3	0.677 ± 0.023	-93 ± 5

IV PREDICTED ISOSPIN AMPLITUDES

The amplitudes in Table III may now be added up to give the contributions to the different $D^0 \rightarrow \rho\pi$ decays. The results are shown in Fig. 5 for the fits in Refs. [7] and [8]. Also shown are the corresponding isospin amplitudes. We give in Tables IV, V, VI, and VII the corresponding numerical values and their estimated errors. One obtains dominance of isospin zero, though not quite as fully as in the BaBar data. The difference appears to be mainly in the $I = 1$ amplitude, whose suppression is not predicted to be as extreme as is observed. It is in this amplitude that the greatest difference is seen between the fits of Refs. [7] and [8].

Table IV: Predicted amplitudes (in units of 10^{-6}) and phases for $D^0 \rightarrow \rho\pi$ in the fit of Ref. [7]. Fit fractions are defined to sum to 100%, and compared to those in Table I normalized to 100% in the last column.

Channel	Amplitude	Phase ($^{\circ}$)	Fraction (%)	vs. BaBar (%)
$\rho^+\pi^-$	1.615 ± 0.058	161 ± 2	55.2 ± 4.0	52.7 ± 0.5
$\rho^0\pi^0$	0.946 ± 0.036	-30 ± 2	18.9 ± 1.4	20.4 ± 0.9
$\rho^-\pi^+$	1.107 ± 0.038	142 ± 2	25.9 ± 1.8	26.9 ± 0.7

Table V: Predicted isospin amplitudes (in units of 10^{-6}) and phases for $D^0 \rightarrow \rho\pi$ in the fit of Ref. [7]. Fit fractions are defined as in Table IV.

Channel	Amplitude	Phase ($^{\circ}$)	Fraction (%)	vs. BaBar (%)
$I = 0$	2.097 ± 0.076	153 ± 1	92.9 ± 6.7	94.24 ± 0.40
$I = 1$	0.477 ± 0.015	-166 ± 2	4.8 ± 0.3	2.17 ± 0.17
$I = 2$	0.330 ± 0.057	162 ± 5	2.3 ± 0.8	3.58 ± 0.29

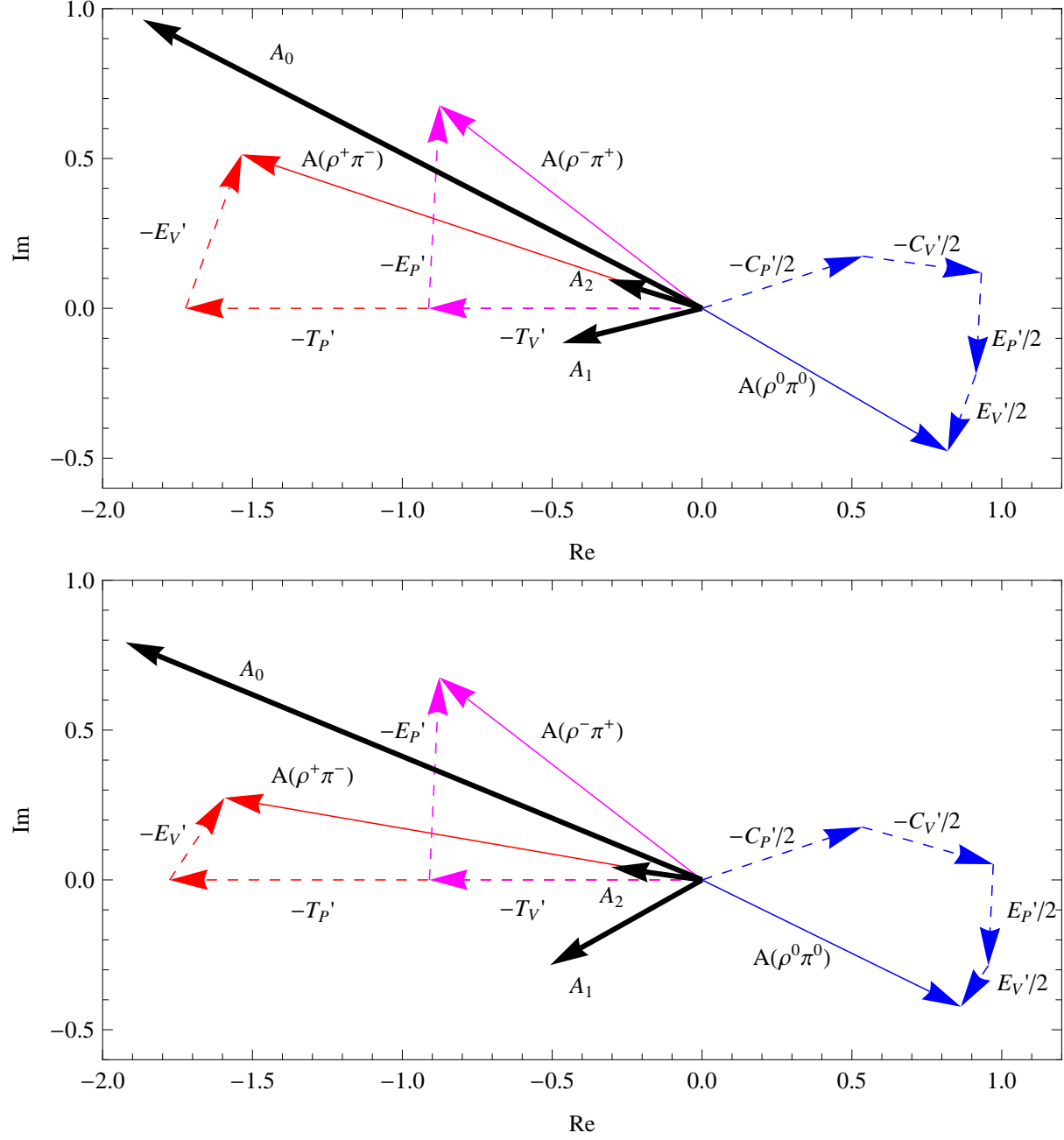


Figure 5: Summing graphical amplitudes to obtain predictions for $D^0 \rightarrow \rho\pi$ and corresponding isospin amplitudes. Top: Ref. [7]; bottom: Ref. [8]

Table VI: Predicted amplitudes (in units of 10^{-6}) and phases for $D^0 \rightarrow \rho\pi$ in the fit of Ref. [8]. Fit fractions are defined as in Table IV.

Channel	Amplitude	Phase ($^\circ$)	Fraction (%)	vs. BaBar (%)
$\rho^+\pi^-$	1.62 ± 0.23	170 ± 7	54.9 ± 15.9	52.7 ± 0.5
$\rho^0\pi^0$	0.96 ± 0.15	-26 ± 12	19.4 ± 6.2	20.4 ± 0.9
$\rho^-\pi^+$	1.11 ± 0.06	142 ± 2	25.7 ± 2.6	26.9 ± 0.7

Table VII: Predicted isospin amplitudes (in units of 10^{-6}) and phases for $D^0 \rightarrow \rho\pi$ in the fit of Ref. [8]. Fit fractions are defined as in Table IV.

Channel	Amplitude	Phase ($^\circ$)	Fraction (%)	vs. BaBar (%)
$I = 0$	2.09 ± 0.21	158 ± 6	90.9 ± 18.2	94.24 ± 0.40
$I = 1$	0.58 ± 0.17	-151 ± 15	7.1 ± 4.1	2.17 ± 0.17
$I = 2$	0.31 ± 0.08	172 ± 27	2.0 ± 1.0	3.58 ± 0.29

V FEATURES OF $I = 0$ DOMINANCE

In the flavor-SU(3) approach one can note several regularities which contribute to the suppression of $I = 1$ and $I = 2$ amplitudes.

(1) The tree amplitudes were assumed in Refs. [7] and [8] to be real and positive, in accord with the expectation from factorization. To the extent that they are equal, their contributions cancel in the $I = 1$ amplitude. Equality would imply relations between form factors and coupling constants; one understands $|T'_P| > |T'_V|$ on the basis of $f_\rho > f_\pi$.

(2) The exchange amplitudes cannot contribute to $I = 2$ and thus their contributions must (and do) cancel exactly.

Some remaining effects seem more accidental:

(3) The exchange amplitudes, having the same phases, add destructively in the $I = 1$ amplitude and constructively in the $I = 0$ amplitude.

(4) The color-suppressed amplitudes approximately cancel the tree amplitudes in the $I = 2$ channel. (They do not contribute at all to $I = 1$ as they only contribute to $\rho^0\pi^0$.)

One might be tempted to blame the enhancement of $I = 0$ on a direct-channel resonance (which would then force the relative phases of various flavor-SU(3) amplitudes to conspire to enhance the $I = 0$ channel). However, a three-pion final state has odd G -parity. If it has $I = 0$ it also must have odd C -parity. But a state with spin-parity-charge-conjugation eigenvalue $J^{PC} = 0^{--}$ is *exotic*, i.e., it cannot be made of a quark-antiquark pair. If such a resonance exists near the D^0 , it must be of an unusual type, such as a hybrid or tetraquark.

VI CONCLUSIONS

The dominance of the isospin-zero channel in $D^0 \rightarrow \pi^+\pi^-\pi^0$ is reproduced in SU(3)-flavor fits to charmed meson decays, with some insight into the suppression of certain contributions to $I = 1$ and $I = 2$ channels. The fact that the relative phase of the $D^0 \rightarrow$

$\rho^+\pi^-$ and $D^0 \rightarrow \rho^-\pi^+$ amplitudes is small supports the idea of factorization whereby the dominant tree (T') amplitudes are in phase with one another. This leads to a cancellation in the $I = 1$ channel. The approximate cancellation of the $D^0 \rightarrow \rho^\pm\pi^\mp$ amplitude by the $D^0 \rightarrow \rho^0\pi^0$ amplitude in the $I = 2$ channel requires a cooperation between color-suppressed (C') and T' amplitudes.

A simple overall explanation of the remarkably simple Dalitz plot structure (with strong depletion along each symmetry axis) remains elusive. It would be interesting to exhibit other Dalitz plots with such simple structure to see if there are some common features we might have overlooked. The invocation of a direct-channel resonance is tempting, but demands an as-yet-unobserved exotic state.

ACKNOWLEDGMENTS

We thank M. Gaspero, K. Mishra, B. Meadows, and A. Soffer for helpful communications. This work was supported in part by the United States Department of Energy through Grant No. DE FG02 90ER40560 and by the National Science Council of Taiwan, R. O. C. under Grant No. NSC 97-2112-M-008-002-MY3. J.L.R. would like to thank the National Center for Theoretical Sciences for extending hospitality.

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